

Lesson 6

Combinational Logic Circuits

Gate Review

- AND Gate
- OR Gate
- NOT Gate
- NAND Gate
- NOR Gate

Gates and Integrated Circuits

- Gates are not sold individually
- They are sold in units called integrated circuits (ICs).
- A chip is a small electronic device consisting of the necessary electronic components (transistors, resistors, and capacitors) to implement various gates.

Logic circuits

- Digital logic chips are combined to produce circuits.
- Logic circuits can be categorized as either
 - combinational logic
 - or sequential logic

This lesson covers combinational logic.

Combinational Logic Circuit

- Combining a number of basic logic gates in a larger circuit to produce more complex logical operations
- A combinational logic circuit consists of logic gates whose outputs at any time are determined directly from the present combination of inputs without regard to previous inputs
- Combinational circuit is a circuit in which we combine the different gates in the circuit, for example encoder, decoder, multiplexer and demultiplexer.
- Combinational logic is used to build circuits that contain basic Boolean operators, inputs, and outputs. In a combinational circuit, an output is always based entirely on the given inputs.

Combinational Logic Circuit

- Let us recall that Boolean algebra allows us to analyze and design digital circuits.
- The output of a combinational circuit is a function of its inputs, and the output is uniquely determined by the values of the inputs at any given time
- The following are some of the combinational circuits that we will discuss: half adders, full adders, multiplexers, demultiplexers, decoders,

Exclusive OR

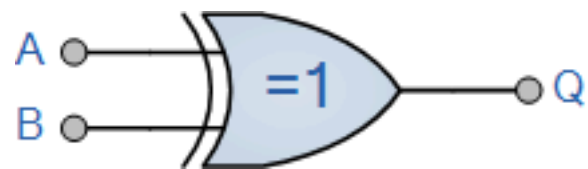
- **Exclusive OR(Ex-OR)** yields true if exactly one (but not both) of two conditions is true.
- we know that for a 2-input OR gate, if $A = 1$, **OR** $B = 1$, OR both $A + B = 1$ then the output from the digital gate must also be at a logic level 1
- If however, a logic output 1 is obtained when only $A = 1$ or when **ONLY** $B = 1$ but **NOT** both together at the same time, giving the binary inputs of 01 or 10, then the output will be 1. This type of gate is known as an Exclusive-OR function

Truth Table for Ex-OR (XOR)

A	B	A EX-OR B
0	0	0
0	1	1
1	0	1
1	1	0

The Boolean expression for A EX-OR B = $AB' + A'B$ (A AND NOT B OR B AND NOT A)

EX-OR Symbol



Draw a circuit for $AB' + A'B$

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Adders

- An adder is a digital circuit that performs addition of numbers.
- In many computers and other processors, adders are used in the arithmetic logic units or ALU. We will cover ALU on Wednesday.
- In this lesson we cover half-adders and full-adders. Before we talk about adders we need to understand binary addition

Review on Binary Addition

- Binary addition is much like your normal everyday addition (decimal addition), except that it carries on a value of 2 instead of a value of 10.
- There are four rules of binary addition:
 - $0 + 0 = 0$
 - $0 + 1 = 1$
 - $1 + 0 = 1$
 - $1 + 1 = 10$ (which is 0 carry 1)

Example 1

- $1 + 11 = ?$

- Solution

$$\begin{array}{r} 1 \\ + 11 \\ \hline \end{array}$$

If we take the first column from the right, we get the binary addition of 1 and 1, which is: $1 + 1 = 10 = 0$ carry 1

The rightmost digit of our answer is therefore 0.

- The second column from the right becomes: $0 + 1 + 1$ (from the carry). In binary addition: $0 + 1 + 1 = 10 = 0$ carry 1
- The second rightmost digit is a 0 and a 1 is carried to the next column. The next column doesn't exist (there are no numbers), therefore the 1 drops into the next slot of the answer. So our answer is: 1 0 0

Let us try the following problems

- $1010 + 11 = ?$

- $100101 + 10101 = ?$

Half-adder

- A half-adder is a very simple combinational logic circuit with two inputs and two outputs
- The half-adder can only add two bits together
- The half adder adds two single binary digits A and B. It has two outputs, sum (S) and carry (C)
- Consider adding two binary digits together: The three things to remember when adding binary digits are:
 - $0 + 0 = 0$
 - $0 + 1 = 1 + 0 = 1$
 - $1 + 1 = 10$.
- In the above there is a sum and carry at the outputs. Sum is an XOR. The Carry output is equivalent to that of an AND gate

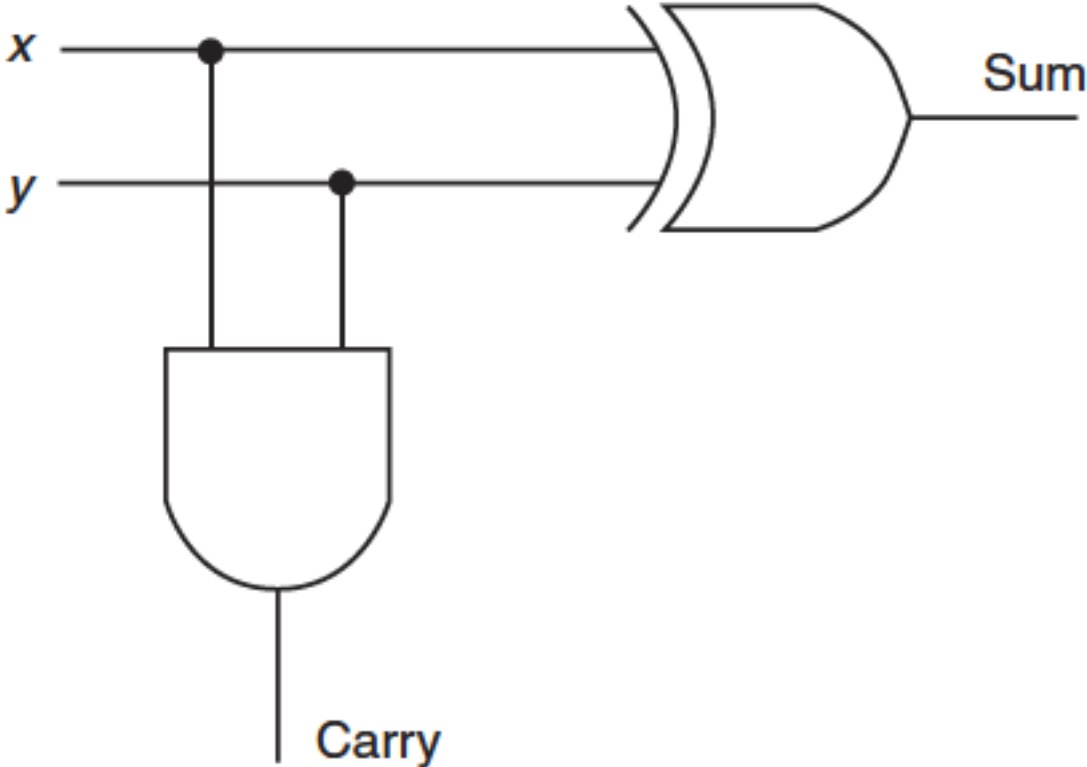
Half-adder

- The simplest half-adder design incorporates an XOR gate for S and an AND gate for C. Refer to the diagram in the next slide for more

Truth Table for a Half-Adder

Inputs		Outputs	
x	y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

The Logic Diagram for a Half-Adder



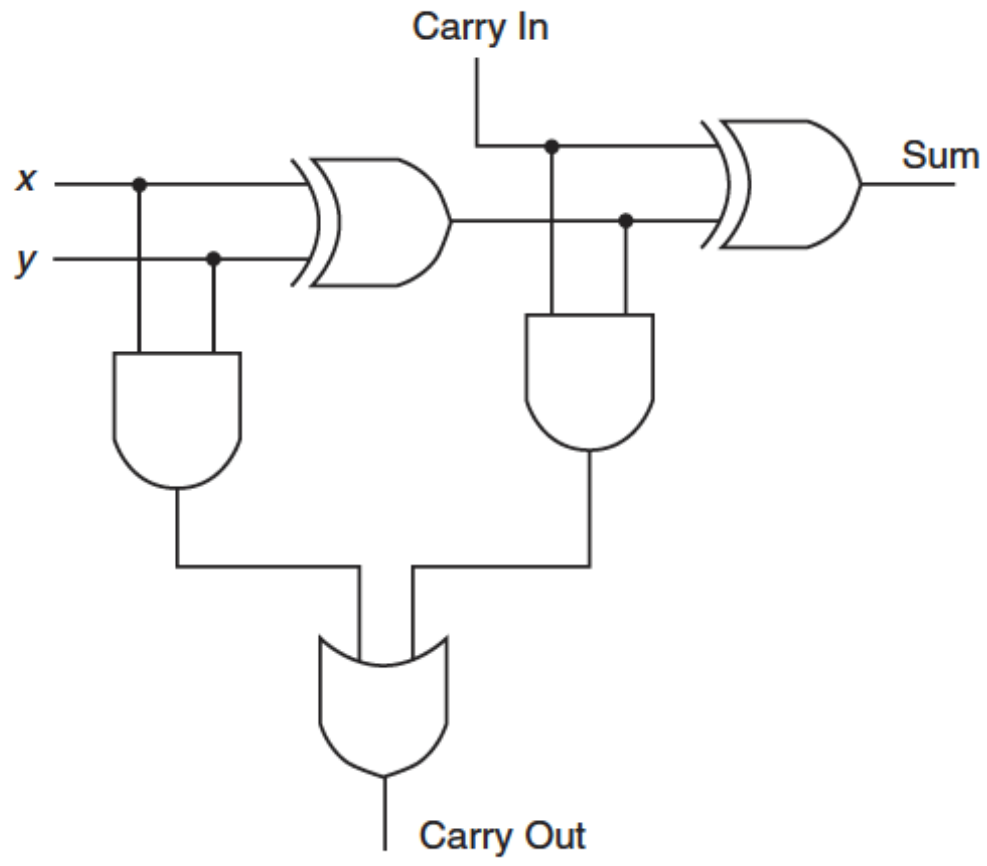
Full-adder

- Full-adder is composed of two half-adders and an OR gate.
- The full-adder is a three input and two output combinational circuit.
- The first two inputs are A and B and the third input is an input carry as C-IN.
- The output carry is designated as C-OUT and the normal output is designated as S which is SUM.
- A full adder adds binary numbers and accounts for values carried in as well as out.

A Truth Table for a Full-Adder

Inputs			Outputs	
x	y	Carry In	Sum	Carry Out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

A Logic Diagram for a Full-Adder



Reading

- Hennessy and Patterson Chapter 8.3 (Appendix B)